# Mechanical Properties of Matter

## Week 10, Lesson 2

- States of Matter
- Density, Hooke's Law & Elasticity
- Specific Gravity, Bulk Modulus & Shear Modulus
- Pressures in Fluids (Fluids at Rest)

References/Reading Preparation:

Schaum's Outline Ch. 12

Principles of Physics by Beuche – Ch.9

# Mechanical Properties of Matter

Introduction:

All materials are composed of atoms. The force between atoms is basically electrical in nature (stemming from the fact that atoms are composed of charged particles).

The way in which these atoms are arranged and in what combination ultimately determines a material's *bulk behavior.*

It is these properties of *bulk matter*, often referred to as mechanical properties, that are of interest.

In this section, we will study mechanical properties such as density, elasticity, and fluid pressure and flow.

# States of Matter

The world around us is composed of three recognizable types of materials: solids, liquids, and gases.

We call these types the three *states of matter*.

The fundamental difference between them lies in the way that forces act between the atoms or molecules composing the substance.

In gases, the interatomic forces are practically nonexistent, allowing individual gas atoms (or molecules) to move independently.

This freedom of motion allows the gas to fill any available volume.

In **liquids and solids** these interatomic forces are strong enough so that external forces cannot significantly change the volume occupied by a given solid or liquid sample.

Thus we say that solids and liquids are nearly *incompressible*.

In **solids**, the interatomic forces arrange the atoms in a rigid three-dimensional array, or structure.

As a result, solids not only are incompressible, but also *rigid* – resisting any attempts to change their shape.

**Liquids** do not have this three-dimensional rigid structure.

Therefore, liquids are ready deformable in shape, conforming to the shape of any container, and are able to flow in response to applied forces.

The state in which a particular substance exists depends on the temperature of the substance and on the external pressure surrounding it.

# Density

We frequently make use of a property called the **density** of a material.

The mass density ($\rho$) of a material is the mass per unit volume of the material:

$$\rho = \frac{\text{mass of body}}{\text{volume of body}} = \frac{m}{V}$$

The SI unit for mass density is $kg/m^3$

Although, $g/cm^3$ is also used, where $1000\ kg/m^3 = 1\ g/cm^3$

The density of water is close to $1000\ kg/m^3$.

# Specific Gravity

A property closely related to density is **specific gravity** (SG).

The **specific gravity** (SG or sp gr) of a substance is the ratio of the density of the substance to the density of some *standard substance*.

The standard is usually water (at 4 °C) for liquids and solids, while for gases, it is usually air.

$$SG = \frac{\rho}{\rho_{standard}}$$

Notice that specific gravity is a *dimensionless number*.

## Worked Example

A cube of uranium ($\rho$ = 18,680 kg/m$^3$) is 2.00 cm on each side.

a) Find its mass.

b) How large a cube of ice ($\rho_{ice}$ = 920 kg/m$^3$) has the same mass?

## Hooke’s Law – Elastic Moduli

Many objects, such as a coiled spring or a metal rod, exhibit a property called **elasticity**.

When stretched or compressed by an applied force, they tend to return to their original length when the force is removed.

An example is that of a spring – as we saw in one of our lab experiments. There is a direct relationship between the change in length and the force applied.

This is stated clearly in Hooke’s Law:

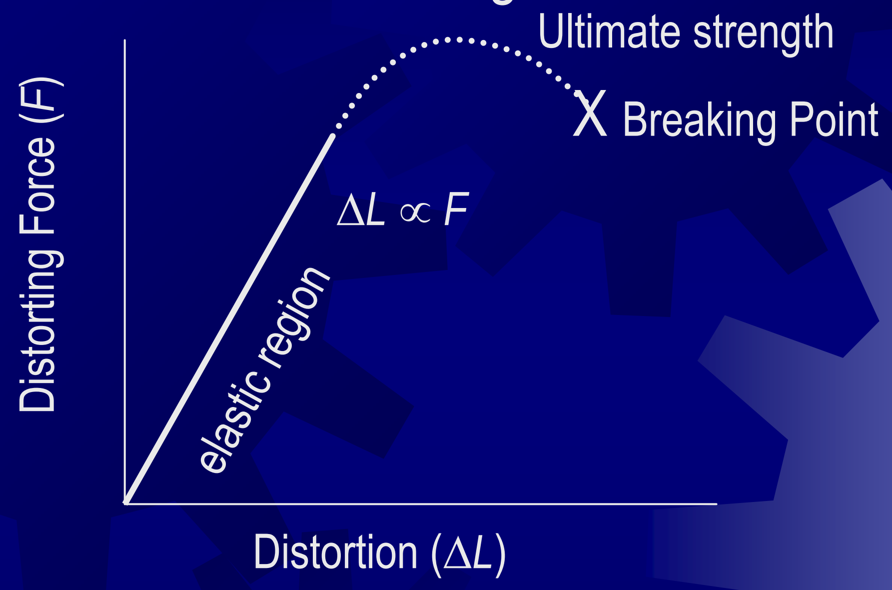
> When an elastic object is stretched or otherwise distorted, the amount of distortion is linearly proportional to the distorting force.

Hooke's Law also applies to objects other than springs – such as a solid rod.

The following illustration shows the behavior of a solid rod to a force.

The straight line portion is called the elastic region.

This is where $\Delta L \propto F$.



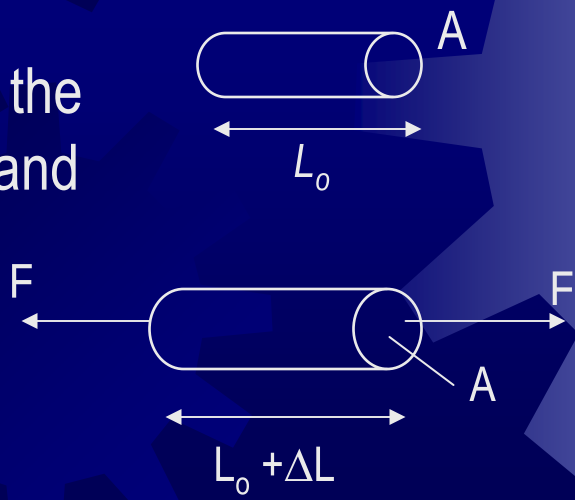

If the rod is stretched too much, beyond the elastic limit, it deviates from this direct proportionality between $\Delta L$ and $F$, and does not return to its original length when the force is removed.

The previous illustration of the behavior of a rod subjected to a force shows that for small deformations, the rod shows *elastic behavior* and obeys Hooke's Law.

To use Hooke's Law in describing the elastic properties of solids we use the terms *stress* and *strain*.

Let's define these quantities in terms of a stretching experiment on a rod with cross-sectional area A.

A tensile force F pulls perpendicularly to the end area A of a rod of original length $L_o$ and causes it to stretch an amount $\Delta L$.



From this illustration we can define stress and strain.

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

(SI units are $N/m^2$)

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L_o}$$

(strain is dimensionless)

# Hooke's Law

Hooke's Law can be stated in terms of stress and strain.

If a system obeys Hooke's Law, then stress $\propto$ strain.

We then define a constant, called the *modulus of elasticity*, by the relation:

$$\boxed{\text{Modulus of elasticity} = \frac{\text{stress}}{\text{strain}}}$$

The modulus has the same units as stress.

A large modulus means that a large stress is required to produce a given strain.

# The Young's Modulus

The Young's Modulus (Y) describes the length elasticity of a material.

Suppose a wire or rod of original length, *L*, and cross-sectional area, A, elongates an amount $\Delta L$, under a force *F* applied to its end.

Then, Tensile stress = $F/A$ and Tensile strain = $\Delta L/L$

and

$$\text{Young's Modulus} = Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L}$$

$$= \frac{FL}{A\,\Delta L} \quad \text{(its SI unit is Pa)}$$

The value of Y depends only on the material of the wire, and NOT on its dimensions.

## Worked example

The specific gravity of cast iron is 7.20. Find its density and the mass of 60 $cm^3$ of it.

(ans. 7200 $kg/m^3$, 0.432 kg)

## Worked example

A drum holds 200 kg of water or 132 kg of gasoline. Determine for the gasoline:

a) Its sp gr
b) $\rho$ in kg/m$^3$

(ans. 0.66, 660 kg/m$^3$)

## Worked example

A load of 50 kg is applied to the lower end of a steel rod 80 cm long and 0.60 cm in diameter. Determine:

a) The stress.
b) The amount the rod stretches (Y = 190 Gpa for steel)
c) The strain.